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Spin Correlations in Nonrelativistic Quantum Mechanics

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Einstein–Podolsky–Rosen spin correlations in the framework of nonrelativistic quantum mechanics for moving observers are calculated. The measurements are performed in bounded regions of space (detectors), not necessarily simultaneously. The resulting correlation function depends not only on the directions of spin measurements but also on the relative velocity of the observers.

KEY WORDS: EPR correlations; EPR paradox; correlation function.

1. INTRODUCTION

The Einstein–Podolsky–Rosen (EPR) paradox (Bohm, 1951; Einstein *et al.*, 1935) and its consequences, like the violation of Bell-type inequalities (see e.g., Apostolakis *et al.*, 1998; Aspect *et al.*, 1981, 1982; Bell, 1964; Tittel *et al.*, 1998, 1999; Weihs *et al.*, 1998), are strictly connected with recent developments in the area of quantum mechanics.

In this paper we analyze the EPR spin correlations in the framework of nonrelativistic quantum mechanics. Let us remind briefly how the standard formula for the correlation function is obtained (see e.g., Peres, 1995). We consider a source which emits two spin 1/2 particles in opposite directions. Two observers, say $\mathcal{A}$ and $\mathcal{B}$, measure the spin component of the particle along given directions, **a** and **b**, respectively. If we assume that the particles are in the singlet state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle), \tag{1}$$

then the correlation function reads

$$\mathcal{C}(\mathbf{a}, \mathbf{b}) = \langle\Psi|\mathbf{a}\cdot\boldsymbol{\sigma} \otimes \mathbf{b}\cdot\boldsymbol{\sigma}|\Psi\rangle = -\mathbf{a}\cdot\mathbf{b} = -\cos\theta, \tag{2}$$

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where **a**, **b**—unit vectors, $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$—Pauli spin matrices, and θ denotes the angle between vectors **a** and **b**.

Notice that in the above considerations one neglects the space degrees of freedom as well as the size of the detectors used by the observers $\mathcal{A}$ and $\mathcal{B}$. Thus the main goal of our analysis is to fill this gap and calculate the correlation function taking into account:

- the space part of the wave function
- the finite size of the detectors
- the relative motion of the observers
- the fact that measurements can be performed at different times.

For simplicity we consider only the case of distinguishable particles. The case of identical particles is considered by Caban *et al.* (in press). The calculation of the correlation function in the Lorentz covariant quantum mechanics is considered by Rembieliński and Smoliński (2002).

2. GALILEAN GROUP AND ITS UNITARY REPRESENTATIONS

Let us begin with a summary of the main facts concerning Galilean group and its unitary ray representations which we will need to take into account the relative motion of the observers $\mathcal{A}$ and $\mathcal{B}$. Let $\mathcal{H}$ denote one-particle Hilbert space of states. The basic observables: $\hat{\mathbf{X}}$—position, $\hat{\mathbf{P}}$—momentum, $\hat{\mathbf{S}}$—spin act in the space $\mathcal{H}$. Their canonical commutation relations are the following:

$$[\hat{X}_i, \hat{X}_j] = 0, \qquad [\hat{P}_i, \hat{P}_j] = 0, \tag{3}$$

$$[\hat{X}_i, \hat{P}_j] = i\delta_{ij}, \qquad [\hat{X}_i, \hat{S}_j] = 0, \tag{4}$$

$$[\hat{P}_i, \hat{S}_j] = 0, \qquad [\hat{S}_i, \hat{S}_j] = i\varepsilon_{ijk}\hat{S}_k. \tag{5}$$

The classical Galilean transformations have the form

$$\mathbf{x}' = R\mathbf{x} + \mathbf{a} - \mathbf{v}t, \qquad t' = t + \tau \tag{6}$$

where **v** denotes the velocity of the frame $(\mathbf{x}', t')$ with respect to the frame $(\mathbf{x}, t)$. In the sequel we adopt the passive point of view. Now we assume that $\mathcal{H}$ is the carrier space of a unitary ray representation of the Galilean group. In the Hilbert space

- rotations are generated by the total angular momentum $\hat{\mathbf{J}}$,
- translations are generated by the momentum $\hat{\mathbf{P}}$,
- time translations are generated by the Hamiltonian $\hat{H}$,
- Galilean boosts are generated by $\hat{\mathbf{G}}$.

The generators of rotations and Galilean boosts can be expressed by basic observables:

$$\hat{\mathbf{J}} = \hat{\mathbf{S}} + \hat{\mathbf{X}} \times \hat{\mathbf{P}}, \qquad \hat{\mathbf{G}} = t\hat{\mathbf{P}} - M\hat{\mathbf{X}}, \tag{7}$$

where M is mass of the system.

In momentum representation the basis vectors of the carrier space of a particular irreducible unitary ray representation of Galilean group in the Schrödinger picture we will denote by $|\mathbf{k}, m\rangle_t$, where $\mathbf{k}$ is an eigenvalue of momentum operator $\hat{\mathbf{P}}$, m—a spin component along the z axis. We will denote the elements of the unitary representation of the Galilean group as follows:

$$U(\mathbf{a}) = e^{i\mathbf{a}\hat{\mathbf{P}}}, \qquad U(\mathbf{v}) = e^{i\mathbf{v}\hat{\mathbf{G}}}, \tag{8}$$

$$U(R) = e^{i\varphi\hat{\mathbf{J}}}, \qquad U(\tau) = e^{i\tau\hat{H}}. \tag{9}$$

The position operator $\hat{\mathbf{X}}$ fulfills the eigenequation:

$$\hat{\mathbf{X}}|\mathbf{x}, m\rangle_t = \mathbf{x}|\mathbf{x}, m\rangle_t, \tag{10}$$

where

$$|\mathbf{x}, m\rangle_t = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{k}\, e^{-ikx} |\mathbf{k}, m\rangle_t. \tag{11}$$

The translations, rotations, and boosts act on vectors $|\mathbf{x}, m\rangle_t$ as follows:

$$U(\mathbf{a})|\mathbf{x}, m\rangle_t = |\mathbf{x} - \mathbf{a}, m\rangle_t, \tag{12}$$

$$U(R)|\mathbf{x}, m\rangle_t = \mathcal{D}^s(R)_{m'm}|R\mathbf{x}, m'\rangle_t, \tag{13}$$

$$U_t(\mathbf{v})|\mathbf{x}, m\rangle_t = e^{iM(\frac{t\mathbf{v}^2}{2} - \mathbf{v}\mathbf{x})}|\mathbf{x} - t\mathbf{v}, m\rangle_t. \tag{14}$$

In discussion of EPR-type experiments it is convenient to use position basis in which vectors are numbered by spin component along arbitrary axis (not necessary z axis). Observable $\mathbf{n} \cdot \hat{\mathbf{S}}$ measures spin component along axis in the direction $\mathbf{n}$. Since $\mathbf{n} \cdot \hat{\mathbf{S}}$ commutes with $\hat{\mathbf{X}}$ these two observables possess common set of eigenvectors. We denote them by $|\mathbf{x}, \mathbf{n}, \lambda\rangle$ and

$$(\mathbf{n} \cdot \hat{\mathbf{S}})|\mathbf{x}, \mathbf{n}, \lambda\rangle = (\mathbf{n} \cdot \mathbf{\Sigma})_{\sigma\lambda}|\mathbf{x}, \mathbf{n}, \sigma\rangle = \lambda|\mathbf{x}, \mathbf{n}, \lambda\rangle, \tag{15}$$

$$\hat{\mathbf{X}}|\mathbf{x}, \mathbf{n}, \lambda\rangle = \mathbf{x}|\mathbf{x}, \mathbf{n}, \lambda\rangle, \tag{16}$$

where $\lambda = -s, -s+1, \ldots, s$ and $\mathbf{\Sigma}$ denotes the generators of the representation $\mathcal{D}^s$. Writting $\mathbf{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ we get

$$|\mathbf{x}, \mathbf{n}, \lambda\rangle = \mathcal{D}^s(e^{i\theta\boldsymbol{\omega}\mathbf{\Sigma}})_{\lambda'\lambda}|\mathbf{x}, \lambda'\rangle_i, \tag{17}$$

where $\boldsymbol{\omega} = (\sin\varphi, -\cos\varphi, 0)$. Therefore taking into account (12)–(17) we get:

$$U(\mathbf{a})|\mathbf{x}, \mathbf{n}, \lambda\rangle = |\mathbf{x} - \mathbf{a}, \mathbf{n}, \lambda\rangle, \tag{18}$$

$$U(R)|\mathbf{x},\mathbf{n},\lambda\rangle = \mathcal{D}^s(R)_{\lambda'\lambda}|R\mathbf{x},R\mathbf{n},\lambda'\rangle, \tag{19}$$

$$U_t(\mathbf{v})|\mathbf{x},\mathbf{n},\lambda\rangle = e^{iM(\frac{t\mathbf{v}^2}{2}-\mathbf{vx})}|\mathbf{x}-t\mathbf{v},\mathbf{n},\lambda\rangle. \tag{20}$$

In particular, when $s = 1/2$ we have

$$\left|\mathbf{x},\mathbf{n},\frac{1}{2}\right\rangle = \cos\frac{\theta}{2}\left|\mathbf{x},\frac{1}{2}\right\rangle + e^{-i\varphi}\sin\frac{\theta}{2}\left|\mathbf{x},-\frac{1}{2}\right\rangle, \tag{21}$$

$$\left|\mathbf{x},\mathbf{n},-\frac{1}{2}\right\rangle = -e^{i\varphi}\sin\frac{\theta}{2}\left|\mathbf{x},\frac{1}{2}\right\rangle + \cos\frac{\theta}{2}\left|\mathbf{x},-\frac{1}{2}\right\rangle. \tag{22}$$

3. CORRELATION FUNCTIONS

We consider two spin s particles, α and β. The space of states of these particles is $\mathcal{H}^\alpha \otimes \mathcal{H}^\beta$ where $\mathcal{H}^\alpha$ and $\mathcal{H}^\beta$—space of states of the particles α and β, respectively. In the spaces $\mathcal{H}^\alpha$ and $\mathcal{H}^\beta$ we will use bases $\{|\mathbf{x}_\alpha,\mathbf{n}_\alpha,\lambda_\alpha\rangle\}$ and $\{|\mathbf{x}_\beta,\mathbf{n}_\beta,\lambda_\beta\rangle\}$, respectively. Recall that vector $|\mathbf{x}_\alpha,\mathbf{n}_\alpha,\lambda_\alpha\rangle$ ($|\mathbf{x}_\beta,\mathbf{n}_\beta,\lambda_\beta\rangle$) describes the situation when the particle $\alpha(\beta)$ is localized at $\mathbf{x}_\alpha$ ($\mathbf{x}_\beta$) and its spin component along the direction determined by a unit vector $\mathbf{n}_\alpha$ ($\mathbf{n}_\beta$) is equal to λ_α (λ_β). We want to describe an EPR-type experiment in which two distant observers $\mathcal{A}$ and $\mathcal{B}$ measure spin components of the particles using detectors which occupy some bounded regions A and B, respectively. Thus the measurement consists of the localization inside the detector and simultaneous measurement of the spin component. Therefore the observers $\mathcal{A}$ and $\mathcal{B}$ measure the observables:

$$\Lambda^s_{A,\mathbf{a}} \otimes I, \qquad I \otimes \Lambda^s_{B,\mathbf{b}}, \tag{23}$$

where the spectral decomposition of $\Lambda^s_{\Omega,\mathbf{n}}(\Omega = A$ or $\Omega = B$, $\mathbf{n} = \mathbf{a}$ or $\mathbf{n} = \mathbf{b})$ is the following

$$\Lambda^s_{\Omega,\mathbf{n}} = \sum_{\lambda=-s}^{s}\lambda\left(\int_\Omega d^3\mathbf{x}|\mathbf{x},\mathbf{n},\lambda\rangle\langle\mathbf{x},\mathbf{n},\lambda|\right) \equiv \sum_{\lambda=-s}^{s}\lambda\Pi^{s,\lambda}_{\Omega,\mathbf{n}}. \tag{24}$$

The projectors $\Pi^{s,\lambda}_{\Omega,\mathbf{n}}$ in (24) have the obvious interpretation: When we measure $\Pi^{s,\lambda}_{\Omega,\mathbf{n}}$ we get the value 1 if and only if the particle is inside Ω and its spin component along the direction $\mathbf{n}$ is equal to λ.

Calculation of the correlation function can be divided in the folowing steps.

- *Preparation of the initial state*
  We assume that a two-particle state ρ is prepared in a certain inertial frame of reference $\mathcal{O}$. Two inertial observers, $\mathcal{A}$ and $\mathcal{B}$, move with constant velocities with respect to $\mathcal{O}$. The velocity of the frame $\mathcal{O}$ with respect to $\mathcal{A}$ and $\mathcal{B}$ we denote by $\mathbf{v}_A$ and $\mathbf{v}_B$, respectively. At time t_A the state ρ is given by $\rho(t_A)$.

- *Measurement performed by observer $\mathcal{A}$*
  For the observer $\mathcal{A}$ the density matrix $\rho(t_A)$ has the form

$$\rho_{\mathcal{A}}(t_A) = U_{t_A}(\mathbf{v}_A)\rho(t_A)U^{\dagger}_{t_A}(\mathbf{v}_A), \tag{25}$$

  where $U_t(\mathbf{v}) = U^{\alpha}_t(\mathbf{v}) \otimes U^{\beta}_t(\mathbf{v})$ and $U^{\alpha}_t(\mathbf{v})$ is the unitary operator of pure Galilean boost. At time t_A the observer $\mathcal{A}$ measures $\Lambda^s_{A,\mathbf{a}} \otimes I$ in the state (25) and as a result of the measurement with selection he receives λ_α with the probability

$$p(\lambda_\alpha) = \mathrm{Tr}\left[\rho_{\mathcal{A}}(t_A)\left(\Pi^{s,\lambda_\alpha}_{A,\mathbf{a}} \otimes I\right)\right] \tag{26}$$

  The measurement reduces the density matrix (25) to

$$\rho^{\lambda_\alpha}_{\mathcal{A}}(t_A) = \frac{\left(\Pi^{s,\lambda_\alpha}_{A,\mathbf{a}} \otimes I\right)\rho_A(t_A)\left(\Pi^{s,\lambda_\alpha}_{A,\mathbf{a}} \otimes I\right)}{\mathrm{Tr}\left[\rho_{\mathcal{A}}(t_A)\left(\Pi^{s,\lambda_\alpha}_{A,\mathbf{a}} \otimes I\right)\right]}. \tag{27}$$

- *Free time evolution of the state*
  The density matrix (27) as seen from the frame $\mathcal{O}$ reads

$$\rho^{\lambda_\alpha}(t_A) = U^{\dagger}_{t_A}(\mathbf{v}_A)\rho^{\lambda_\alpha}_{\mathcal{A}}(t_A)U_{t_A}(\mathbf{v}_A). \tag{28}$$

  Now the state $\rho^{\lambda_\alpha}(t_A)$ evolves from time t_A to t_B and resulting density matrix reads

$$\rho^{\lambda_\alpha}(t_B) = U^{\dagger}(t_B - t_A)\rho^{\lambda_\alpha}(t_A)U(t_B - t_A), \tag{29}$$

  where $U(t_B - t_A) = U^{\alpha}(t_B - t_A) \otimes U^{\beta}(t_B - t_A)$ and $U(t)$—the time evolution operator.
- *Measurement performed by observer $\mathcal{B}$*
  The density matrix (29) as seen by the observer $\mathcal{B}$ has the form

$$\rho^{\lambda_\alpha}_{\mathcal{B}}(t_B) = U_{t_B}(\mathbf{v}_B)\rho^{\lambda_\alpha}(t_B)U^{\dagger}_{t_B}(\mathbf{v}_B). \tag{30}$$

  At time t_B the observer $\mathcal{B}$ measures $I \otimes \Lambda^s_{B,\mathbf{b}}$ in the state (30) and receives λ_β with the probability

$$p(\lambda_\beta|\lambda_\alpha) = \mathrm{Tr}\left[\rho^{\lambda_\alpha}_{\mathcal{B}}(t_B)\left(I \otimes \Pi^{s,\lambda_\beta}_{B,\mathbf{b}}\right)\right]. \tag{31}$$

  It is conditional probability because the state in which $\mathcal{B}$ performs the measurement has the form (29) only if $\mathcal{A}$ receives λ_a in the first measurement.

Correlation function is defined by the following formula

$$\mathcal{C}^{\alpha,\beta}(\mathbf{a}, \mathbf{b}) = \sum_{\lambda_\alpha,\lambda_\beta} \lambda_\alpha\lambda_\beta\, p(\lambda_\alpha, \lambda_\beta), \tag{32}$$

where $p(\lambda_\alpha, \lambda_\beta)$ denotes the probability that $\mathcal{A}$ receives λ_α and $\mathcal{B}$ receives λ_β.

We have

$$p(\lambda_\alpha, \lambda_\beta) = p(\lambda_\alpha) p(\lambda_\beta|\lambda_\alpha), \tag{33}$$

so, taking into account (32), (26), (31), we get

$$\mathcal{C}^{\alpha\beta}(\mathbf{a}, \mathbf{b}) = \mathrm{Tr}\big\{\rho(t_A)\big[U_{t_A}^{\alpha\dagger}(\mathbf{v}_A)\Lambda^s_{A,\mathbf{a}}U^\alpha_{t_A}(\mathbf{v}_A)\big] \otimes \big[U^\beta(t_B - t_A)U_{t_B}^{\beta\dagger}(\mathbf{v}_B) \times \Lambda^s_{B,\mathbf{b}}U^\beta_{t_B}(\mathbf{v}_B)U^{\beta\dagger}(t_B - t_A)\big]\big\}. \tag{34}$$

Now let us take a closer look at the formula (34) in the simplest case. Thus let us assume that

- $p(t_A)$ is a pure state so $\rho(t_A) = |\psi\rangle\langle\psi|$ where $|\psi\rangle \in \mathcal{H}^\alpha \otimes \mathcal{H}^\beta$ is normalized,
- both measurements are performed simultaneously $t_A = t_B = t$.

In the position representation $|\psi\rangle$ reads

$$|\psi\rangle = \sum_{m_\alpha, m_\beta} \int\int d^3\mathbf{x} d^3\mathbf{y}\, \psi_{m_\alpha m_\beta}(\mathbf{x}, \mathbf{y})|\mathbf{x}, m_\alpha\rangle \otimes |\mathbf{y}, m_\beta\rangle. \tag{35}$$

Therefore we get

$$\mathcal{C}^{\alpha\beta}_\psi(\mathbf{a}, \mathbf{b}) = \int_A d^3\mathbf{x} \int_B d^3\mathbf{y} \sum_{\substack{m_\alpha, m_\beta \\ m'_\alpha, m'_\beta}} (\mathbf{a}\cdot\mathbf{S})_{m_\alpha m'_\alpha}(\mathbf{b}\cdot\mathbf{S})_{m_\beta m'_\beta} \times \psi^*_{m'_\alpha m'_\beta}(\mathbf{x} - \mathbf{v}_A t, \mathbf{y} - \mathbf{v}_B t)\psi_{m_\alpha m_\beta}(\mathbf{x} - \mathbf{v}_A t, \mathbf{y} - \mathbf{v}_B t). \tag{36}$$

Now let us apply the formula (36) in the case $s = 1/2$. We cosider separately the cases when the state $|\psi\rangle$ is a singlet or a triplet.

3.1. Singlet State

For the singlet state we have

$$\psi_{m_\alpha m_\beta}(\mathbf{x}, \mathbf{y}) = -\psi_{m_\beta m_\alpha}(\mathbf{x}, \mathbf{y}). \tag{37}$$

Thus from (36) we receive

$$\mathcal{C}^{\alpha\beta}_{\psi_{\mathrm{singlet}}}(\mathbf{a}, \mathbf{b}) = -\frac{1}{2}\cos(\theta_{ab}) \int_A d^3\mathbf{x} \int_B d^3\mathbf{y} |\psi_{\mathrm{singlet}}(\mathbf{x} - \mathbf{v}_A t, \mathbf{y} - \mathbf{v}_B t)|^2, \tag{38}$$

where θ_{ab} denotes an angle between vectors **a** and **b**, $\psi_{\mathrm{singlet}}(\mathbf{x}, \mathbf{y}) \equiv \psi_{\frac{1}{2}, -\frac{1}{2}}(\mathbf{x}, \mathbf{y})$ and the normalization yields

$$\iint d^3\mathbf{x} d^3\mathbf{y} |\psi_{\mathrm{singlet}}(\mathbf{x}, \mathbf{y})|^2 = \frac{1}{2}. \tag{39}$$

If we compare (38) with the standard formula (2) we find, that the only difference is the presence of the factor

$$\int_A d^3\mathbf{x} \int_B d^3\mathbf{y} |\psi_{\text{singlet}}(\mathbf{x} - \mathbf{v}_A t, \mathbf{y} - \mathbf{v}_B t)|^2 \tag{40}$$

which influences the intensity of the correlations.

3.2. Triplet State

For the triplet state we have

$$\psi_{m_\alpha m_\beta}(\mathbf{x}, \mathbf{y}) = \psi_{m_\beta m_\alpha}(\mathbf{x}, \mathbf{y}) \tag{41}$$

and from (36) we get

$$\begin{aligned} \mathcal{C}^{\alpha\beta}_{\psi_{\text{triplet}}}(\mathbf{a}, \mathbf{b}) = \frac{1}{4} \int_A d^3\mathbf{x} \int_B d^3\mathbf{y} \{ & (|\psi_{++}|^2 + |\psi_{--}|^2) \cos\theta_a \cos\theta_b \\ & + \left(\psi^*_{++}\psi_{--}\, e^{i(\varphi_a+\varphi_b)} + \psi^*_{--}\psi_{++}\, e^{-i(\varphi_a+\varphi_b)}\right) \sin\theta_a \sin\theta_b \\ & + (\psi^*_{++}\psi_{+-} - \psi^*_{+-}\psi_{--})(\cos\theta_a \sin\theta_b e^{i\varphi_b} + \sin\theta_a \cos\theta_b\, e^{i\varphi_a}) \\ & + (\psi^*_{+-}\psi_{++} - \psi^*_{--}\psi_{+-})(\cos\theta_a \sin\theta_b\, e^{-i\varphi_b} + \sin\theta_a \cos\theta_b\, e^{-i\varphi_a}) \\ & - 2\psi^*_{+-}\psi_{+-}(\cos\theta_a \cos\theta_b - \sin\theta_a \sin\theta_b \cos(\varphi_a - \varphi_b))\}, \end{aligned} \tag{42}$$

where

$$\psi_{++} = \psi_{\frac{1}{2},\frac{1}{2}}(\mathbf{x} - \mathbf{v}_A t, \mathbf{y} - \mathbf{v}_B t), \tag{43}$$

$$\psi_{+-} = \psi_{\frac{1}{2},-\frac{1}{2}}(\mathbf{x} - \mathbf{v}_A t, \mathbf{y} - \mathbf{v}_B t), \tag{44}$$

$$\psi_{--} = \psi_{-\frac{1}{2},-\frac{1}{2}}(\mathbf{x} - \mathbf{v}_A t, \mathbf{y} - \mathbf{v}_B t), \tag{45}$$

$$\mathbf{a} = (\cos\varphi_a \sin\theta_a, \sin\varphi_a \sin\theta_a, \cos\theta_a), \tag{46}$$

$$\mathbf{b} = (\cos\varphi_b \sin\theta_b, \sin\varphi_b \sin\theta_b, \cos\theta_b), \tag{47}$$

and the normalization yields

$$\iint d^3\mathbf{x} d^3\mathbf{y} \left\{ \left|\psi_{\frac{1}{2},\frac{1}{2}}(\mathbf{x}, \mathbf{y})\right|^2 + \left|\psi_{-\frac{1}{2},-\frac{1}{2}}(\mathbf{x}, \mathbf{y})\right|^2 + 2\left|\psi_{\frac{1}{2},-\frac{1}{2}}(\mathbf{x}, \mathbf{y})\right|^2 \right\} = 1. \tag{48}$$

Note, that the triplet correlation function (42) depends on velocities of frames in more nontrivial way than in the singlet case.

4. CONCLUSIONS

In this paper, we have presented the calculation of the spin correlation functions in the EPR type experiments in the framework of nonrelativistic quantum mechanics. In comparison to the standard derivation we additionally considered the space part of the wave function and the relative motion of the observers. We also took into account the fact that every measurement of the spin component is connected with the simultaneous localization of the particle inside the detector. In the most interesting case of the singlet state of two spin 1/2 particles we received, as one could expect, that the correlation function depends on the vectors **a** and **b** in the standard way. The only difference is the presence of the factor which influences the intensity of the correlations. The triplet correlation function depends on velocities of frames in more nontrivial way than in the singlet case.

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